**EXERCISES [MAI 3.8]**

**TRIGONOMETRIC FUNCTIONS**

**SOLUTIONS**

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## A. Paper 1 questions (SHORT)

**1.**

| Function | Amplitude | Period | Range |
|---|---|---|---|
| $f(x)=\sin x$ | 1 | $2\pi$ | $-1\le y\le 1$ |
| $f(x)=\cos x$ | 1 | $2\pi$ | $-1\le y\le 1$ |
| $f(x)=\sin x+1$ | 1 | $2\pi$ | $0\le y\le 2$ |
| $f(x)=\sin x-1$ | 1 | $2\pi$ | $-2\le y\le 0$ |
| $f(x)=5\sin x$ | 5 | $2\pi$ | $-5\le y\le 5$ |
| $f(x)=-7\sin x$ | 7 | $2\pi$ | $-7\le y\le 7$ |
| $f(x)=\sin 4x$ | 1 | $\pi/2$ | $-1\le y\le 1$ |
| $f(x)=-\cos 4x$ | 1 | $\pi/2$ | $-1\le y\le 1$ |
| $f(x)=3\sin 4x$ | 3 | $\pi/2$ | $-3\le y\le 3$ |
| $f(x)=3\sin 4x+10$ | 3 | $\pi/2$ | $7\le y\le 13$ |
| $f(x)=3\sin 4x-2$ | 3 | $\pi/2$ | $-5\le y\le 1$ |
| $f(x)=-5\sin 3x$ | 5 | $2\pi/3$ | $-1\le y\le 1$ |
| $f(x)=-5\sin x+10$ | 5 | $2\pi$ | $5\le y\le 15$ |
| $f(x)=-5\sin x-10$ | 5 | $2\pi$ | $-15\le y\le -5$ |
| $f(x)=-5\sin x-10$ | 5 | $2\pi$ | $-15\le y\le -5$ |

**2.** (a) amplitude = 80, central value = 100, period = π/2

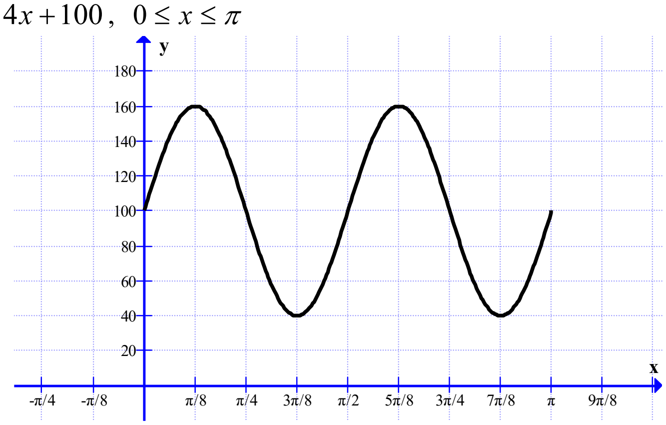
(b) $f(x)=80\sin 4x+100$, since $B=\dfrac{2\pi}{Period}=\dfrac{2\pi}{\pi/2}=4$

(c) (i) $f(x)=-80\sin 4(x-\dfrac{\pi}{4})+100$, ($D=\dfrac{\pi}{4}$ is the position of the 2nd (↓) root)

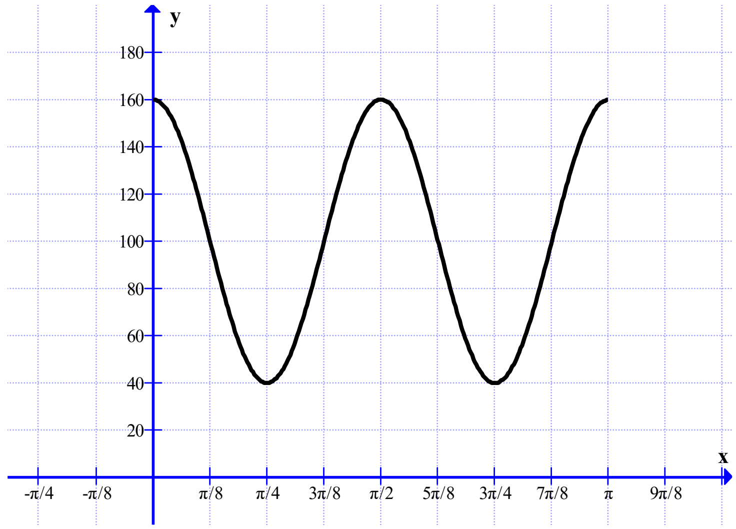
(ii) $f(x)=80\cos 4(x-\dfrac{\pi}{8})+100$, ($D=\dfrac{\pi}{8}$ is the position of the maximum)

(iii) $f(x)=-80\cos 4(x-\dfrac{3\pi}{8})+100$ ($D=\dfrac{3\pi}{8}$ is the position of the minimum)

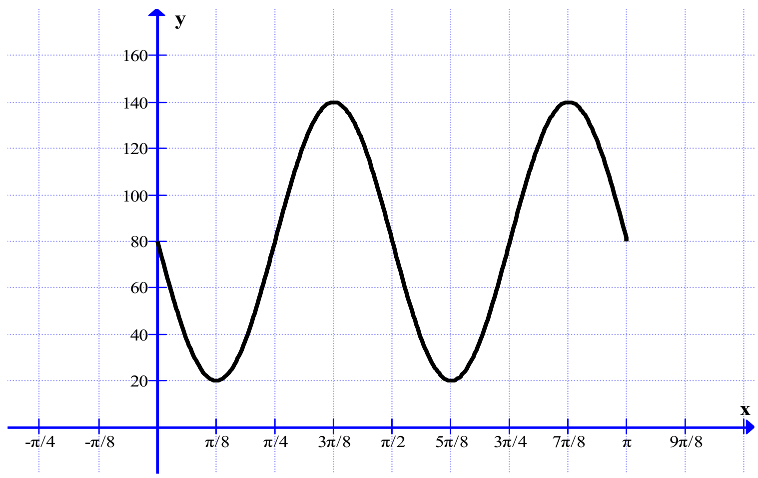
**3.** $f(x)=60\sin 4x+100$, $0\le x\le\pi$



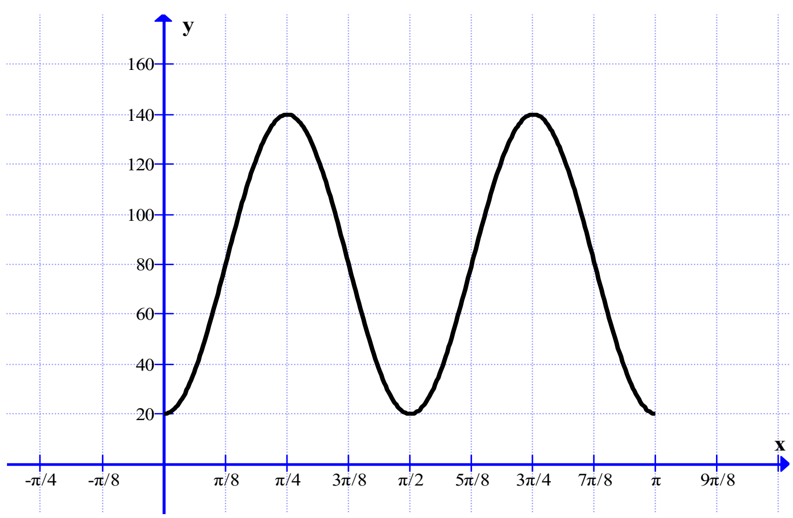
**4.** $f(x) = 60\cos 4x + 100\,,\ \ 0 \le x \le \pi$




**5.** $f(x) = -60\sin 4x + 80\,,\ \ 0 \le x \le \pi$




**6.** $f(x) = -60\cos 4x + 80\,,\ \ 0 \le x \le \pi$




(i) $k = 140$ (ii) $k = 20$ (iii) $20 < k < 140$ (iv) $k < 20$ or $k > 140$

**7.** From sketch of graph $y = 4\sin\left(3x + \frac{\pi}{2}\right)$

or by observing $|\sin \theta| \le 1$. $k > 4, k < -4$

**8.** (a) (i) amplitude = $\frac{7+3}{2} = 5 \Rightarrow p = -5$

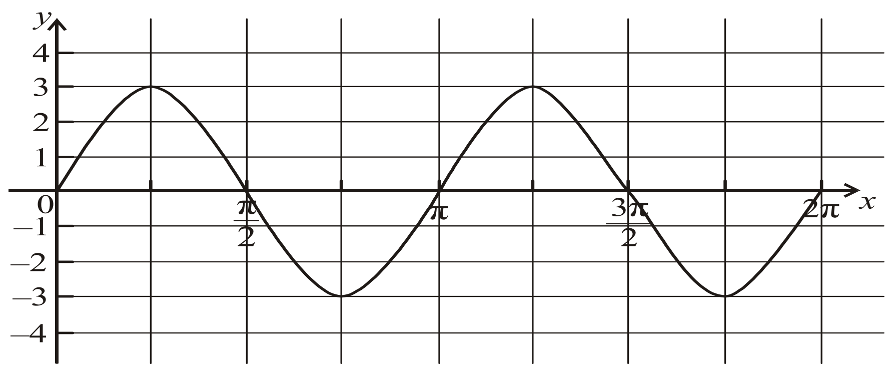
(ii) period = 8 $\Rightarrow$ $q = 0.785 \left(= \frac{2\pi}{8} = \frac{\pi}{4}\right)$

(iii) $r = \frac{7-3}{2} \Rightarrow r = 2$

(b) $k = -3$ (accept $y = -3$)

**9.** (a) period = $\pi$

(b)




(c) 4 (solutions) (intersection points with line $y = 2$ on graph)

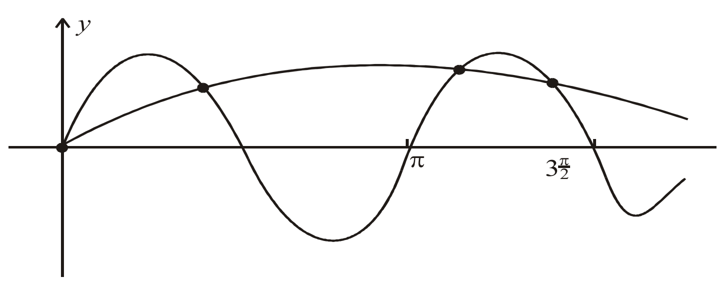
**10.** (a) $p = 30$

(b) Period = $\frac{2\pi}{q} = \frac{\pi}{2} \Rightarrow q = 4$

**11.** (a) (i) –1

(ii) $4\pi$ (accept 720°)

(b)




number of solutions: 4

**12.** $3 = p + q \cos 0 \Rightarrow 3 = p + q$

$-1 = p + q \cos \pi \Rightarrow -1 = p - q$

(i) $p = 1$

(ii) $q = 2$

**13.** **METHOD 1**

The value of cosine varies between –1 and +1. Therefore:

$t = 0 \Rightarrow a + b = 14.3$

$t = 6 \Rightarrow a - b = 10.3$

$\Rightarrow a = 12.3 \quad b = 2$

Period = 12 hours $\Rightarrow \dfrac{2\pi(12)}{k} = 2\pi \Rightarrow k = 12$

**METHOD 2**

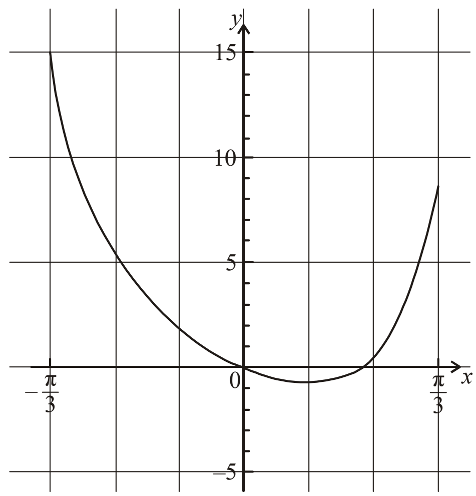
From graph: Midpoint = $a = 12.3$

Amplitude = $b = 2$

Period = $\dfrac{2\pi}{\frac{2\pi}{k}} = 12 \Rightarrow k = 12$

**14.** $a = 4$, $b = 2$, $c = \dfrac{\pi}{2}\left(\text{or } \dfrac{3\pi}{2}\ etc\right)$

**15.** (a)

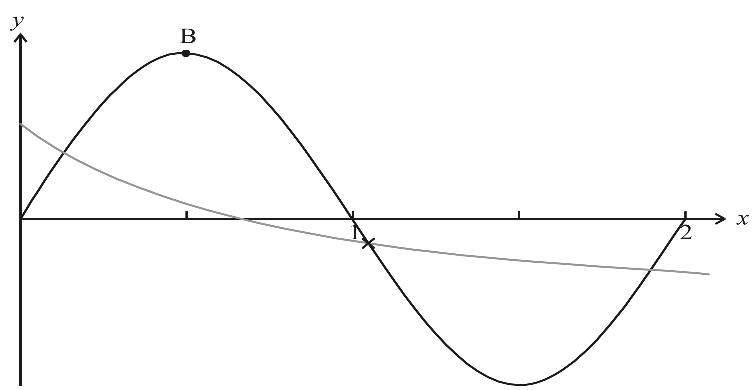



*passing through (0, 0), range approximately –1 to 15.*

(b) $x = -0.207 \quad x = 0.772$

**16.** (a) $b = 6$

(b)




(c) $x = 1.05$ (accept $(1.05, -0.896)$ ) (no additional solutions)

## B. Paper 2 questions (LONG)

**17.** (a) (i) $Q = \frac{1}{2}(14.6 - 8.2) = 3.2$

(ii) $P = \frac{1}{2}(14.6 + 8.2) = 11.4$

(b) $10 = 11.4 + 3.2\cos\left(\frac{\pi}{6}t\right)$

$t = 3.8648$. $t = 3.86$(3 s.f.)

(c) (i) By symmetry, next time is $12 - 3.86... = 8.135...$ $t = 8.14$ (3 s.f.)

(ii) From above, first interval is $3.86 < t < 8.14$

This will happen again, 12 hours later, so $15.9 < t < 20.1$

**18.** (a) (i) 7

(ii) 1

(iii) 10

(b) (i) $A = \frac{18-2}{2} = 8$

(ii) $C = 10$

(iii) period = 12 $B = \frac{\pi}{6}$ (accept 0.524 or 30)

(c) $t = 3.52$, $t = 10.5$, between 03:31 and 10:29 (accept 10:30)

**19.** (a) $f(1) = 3$ $f(5) = 3$

(b) **EITHER** distance between successive maxima = period = 5 – 1 = 4

**OR** period = $\frac{2\pi}{\frac{\pi}{2}} = 4$

(c) **EITHER** $A\sin\left(\frac{\pi}{2}\right) + B = 3$ and $A\sin\left(\frac{3\pi}{2}\right) + B = -1$

$\Leftrightarrow A + B = 3, -A + B = -1$

$\Leftrightarrow A = 2, B = 1$

**OR** Amplitude = $A = \frac{3-(-1)}{2} = \frac{4}{2} = 2$

Midpoint value = $B = \frac{3+(-1)}{2} = \frac{2}{2} = 1$

(d) $f(x) = 2 \Rightarrow 2\sin\left(\frac{\pi}{2}x\right) + 1 = 2 \Rightarrow x = \frac{1}{3}$ or $\frac{5}{3}$ or $\frac{13}{3}$

(e) (i) $k = -1$ (ii) $1 \le k < 3$ (iii) $-1 < k < 1$ or $k = 3$ (iv) $k < -1$ or $k > 3$

**20.** (a) (i) $10 + 4\sin 1 = 13.4$

(ii) At 2100, $t = 21$

$10 + 4\sin 10.5 = 6.48$

(b) (i) 14 metres

(ii) $14 = 10 + 4\sin\left(\frac{t}{2}\right) \Rightarrow t = \pi$ (=3.14)

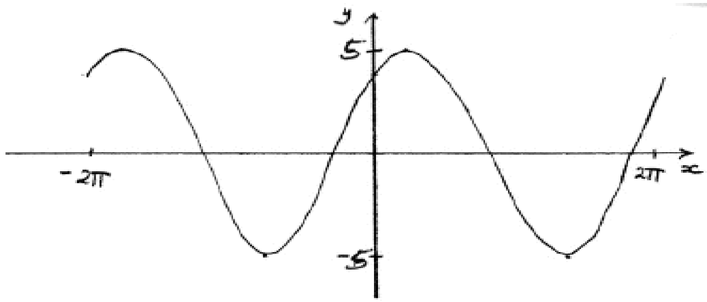
(c) (i) 4

(ii) $10 + 4\sin\left(\frac{t}{2}\right) = 7 \Rightarrow t = 7.98$

(iii) depth < 7 from 8 –11 = 3 hours, from 2030 – 2330 = 3 hours

therefore, total = 6 hours

**21.** (a)




(b) (i) 5 (ii) $2\pi$ (6.28) (iii) $-0.927$

(c) $f(x) = 5 \sin (x + 0.927)$ (accept $p = 5$, $q = 1$, $r = 0.927$)

(d) 3 s.f. values which round to $-5.6$, $0.64$

(e) $k = -5$, $k = 5$

**22.** (a) When $t = 1$, $l = 33 + 5 \cos 4\pi = 38$

(b) (i) $l_{\min} = 33 - 5 = 28$ (ii) $l_{\max} = 33 + 5 = 38$

(c) $33 = 33 + 5\cos 4\pi t \Rightarrow t = 1/8$

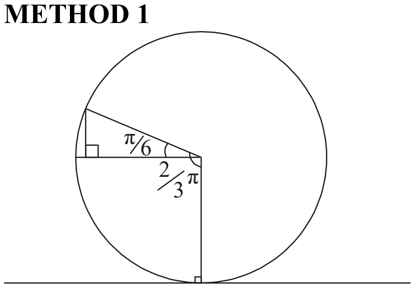
(d) period $= \dfrac{2\pi}{4\pi} = \dfrac{1}{2}$

(e) (i) 10 times (ii) 20 times

**23.** (a) arc AB $= \boldsymbol{r}\theta = 7.85$ (m)

(b) Area of sector AOB $A = \dfrac{1}{2} r^2 \theta = 58.9$ (m$^2$)

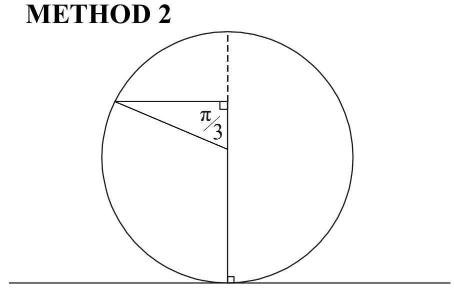
(c) **METHOD 1**




angle $= \dfrac{\pi}{6}$ $(30°)$

height $= 15 + 15 \sin \dfrac{\pi}{6} = 22.5$ (m)

**METHOD 2**




angle $= \dfrac{\pi}{3}$ $(60°)$

height $= 15 + 15 \cos \dfrac{\pi}{3} = 22.5$ (m)

(d) (i) $h\left(\dfrac{\pi}{4}\right) = 15 - 15\cos\left(\dfrac{\pi}{2} + \dfrac{\pi}{4}\right) = 25.6$ (m)

(ii) $h(0) = 15 - 15 \cos \left(0 + \dfrac{\pi}{4}\right) = 4.39$(m)

(iii) **METHOD 1**
Highest point when $h = 30$

$30 = 15 - 15 \cos \left(2t + \dfrac{\pi}{4}\right) \Leftrightarrow t = 1.18 \left(\text{accept } \dfrac{3\pi}{8}\right)$

**METHOD 2**
Using graph $t = 1.18$

**24.** (a) (i) 100 (metres)

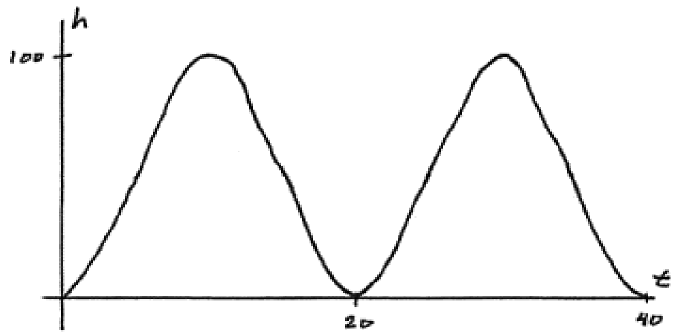
(ii) 50 (metres)

(b) (i) Symmetry with $h(2) = 9.5$

$h(8) = 100 - 9.5 = 90.5$

(ii) $h(21) = h(1) = 2.4$

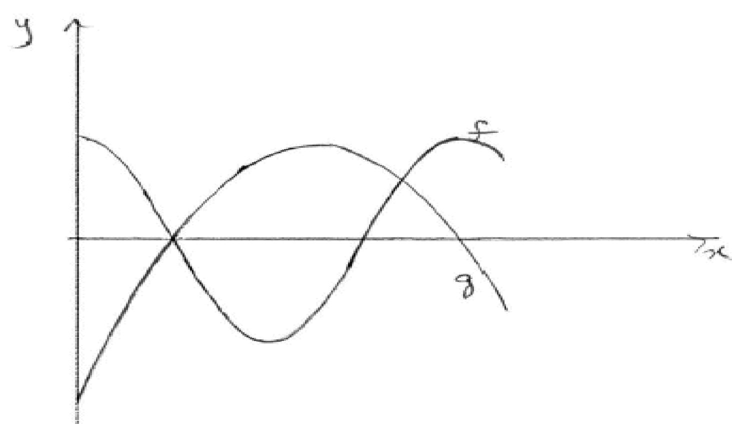
(c)




(d) $b = \dfrac{2\pi}{20}\left(= \dfrac{\pi}{10}\right)$ (accept $b$ =18 if working in degrees)

$a = -50$, $c = 50$

**25.** (a)




(b) (i) (2,0) (or $x = 2$)

(ii) period = 8

(iii) amplitude = 5

(c) (i) (2, 0), (8, 0) (or $x = 2$, $x = 8$)

(ii) $x = 5$ (must be an equation)

(d) intersect when $x = 2$ and $x = 6.79$